



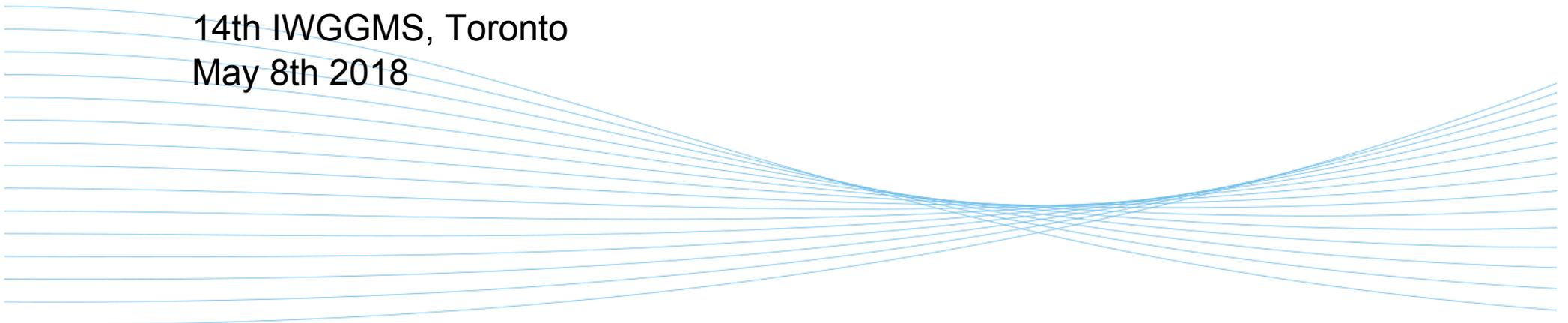
FINNISH METEOROLOGICAL INSTITUTE

Vertical Distribution of Arctic Methane from Ground- based FTS Measurements

Otto Lamminpää, T. Karppinen, S. Tukiainen, M. Laine, J. Tamminen

14th IWGGMS, Toronto

May 8th 2018

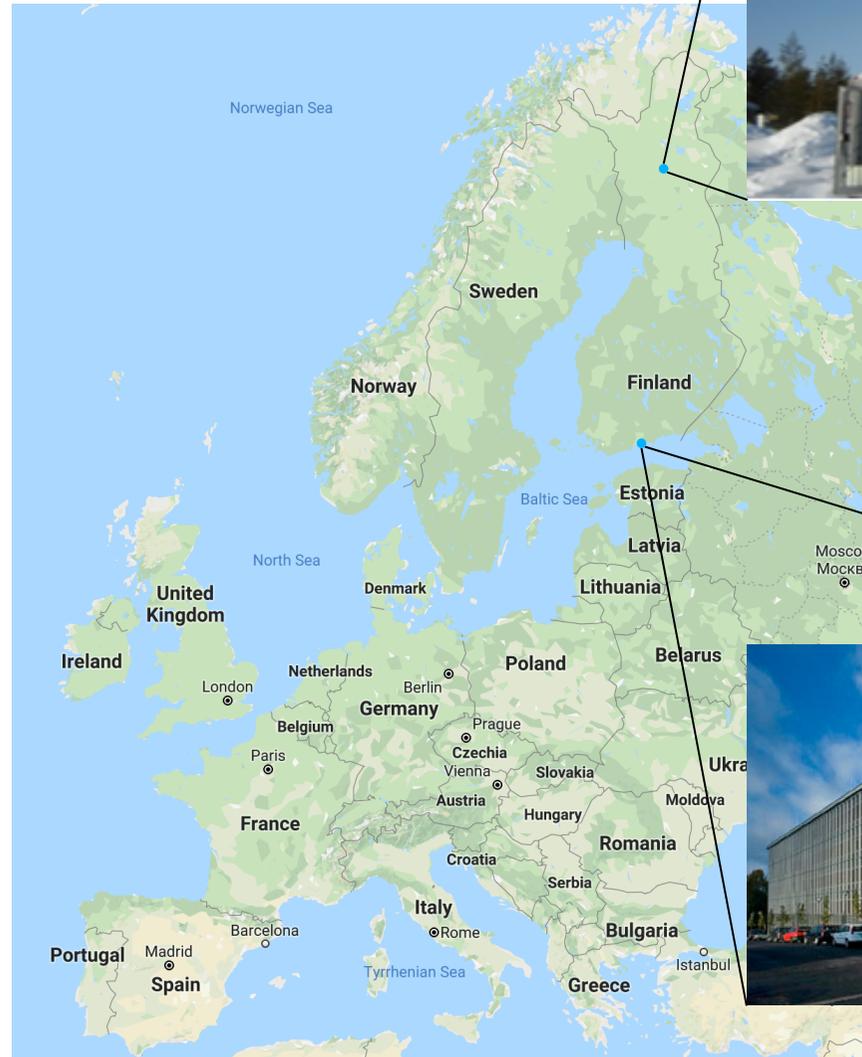




Introduction

Finnish Meteorological Institute
-Sodankylä & Helsinki

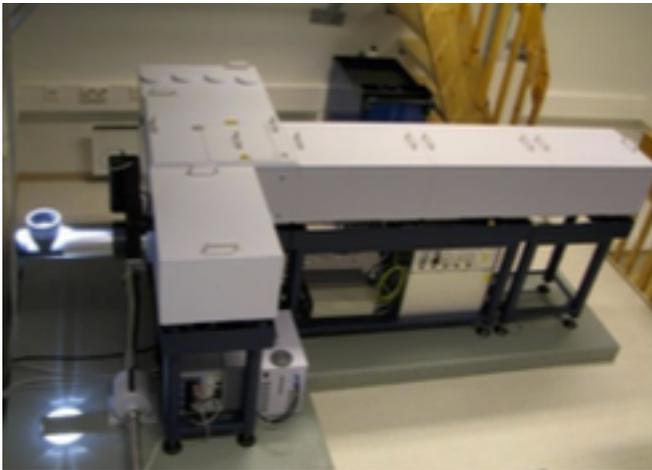
**Greenhouse Gases and
Satellite Methods group**





Introduction: Sodankylä Arctic Research Centre

- Fourier transform infrared spectrometer (FTS): measurement of absorption spectra, part of TCCON network. Our focus: measured **CH₄** [1]
- AirCore balloon sounding: collects gas samples for up to 30km, used as "ground truth" to validate the retrievals [2]



FTIR-spectrometer



AirCore balloon sounding



Introduction: FTS measurement

FTS measurement is modeled with Beer-Lambert law:

$$I(\lambda) = I_0(\lambda) \exp \left(- \sum_i^N \int_l \sigma_i(l) x_i(l) dl \right) (a\lambda^2 + b\lambda + c) + \text{offset}$$



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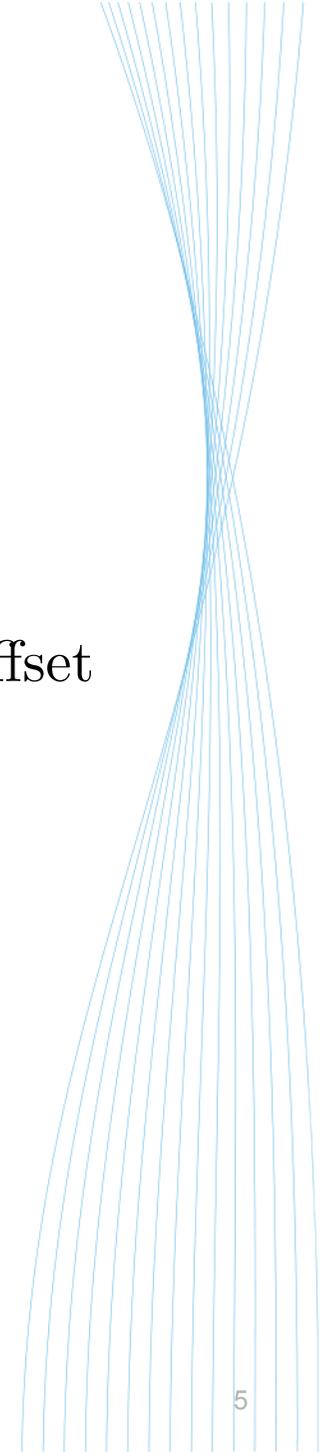
$$I(\lambda) = I_0(\lambda) \exp \left(- \sum_i^N \int_l \sigma_i(l) x_i(l) dl \right) (a\lambda^2 + b\lambda + c) + \text{offset}$$

where for each trace gas i :

$I(\lambda)$ is the intensity of measured light at given wavelength

σ_i are the absorption coefficients

x_i are the unknown trace gas densities

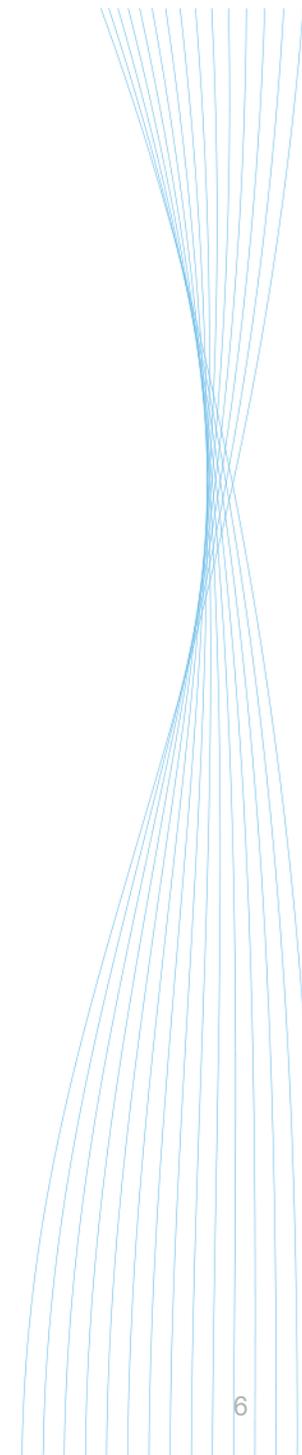




The inverse problem

Consider the non-linear *inverse problem*

$$y = F(x) + \varepsilon$$





The inverse problem

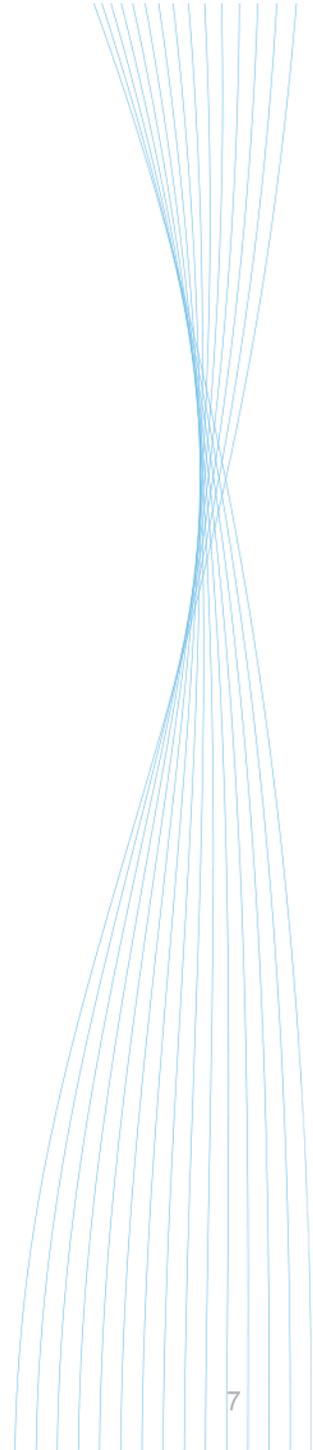
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$$y = F(x) + \varepsilon$$

Statistical approach:

- view all parameters as random variables
- use Bayes' Formula to find the posterior distribution of the unknown \boldsymbol{x} :

$$\pi(\boldsymbol{x}|y) \propto \pi_{\varepsilon}(y|\boldsymbol{x})\pi_{pr}(\boldsymbol{x})$$





The inverse problem

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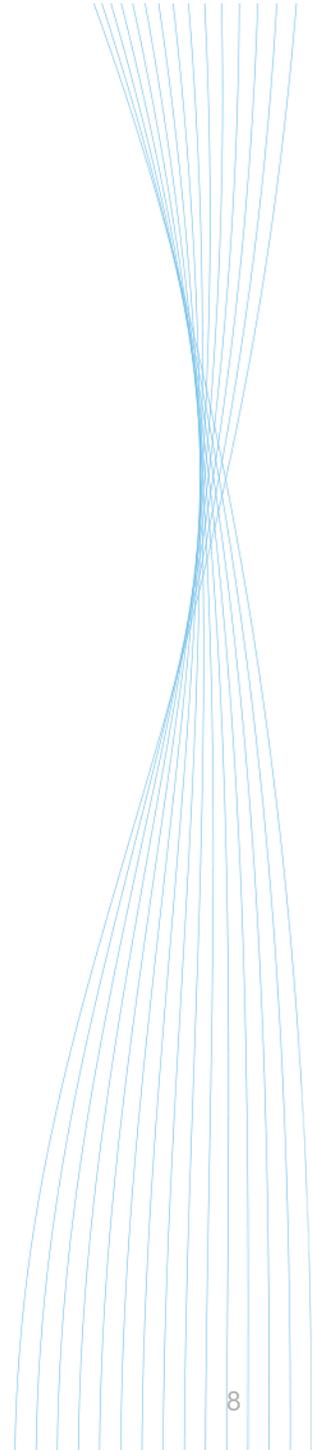
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$$\pi(\boldsymbol{x}|y) \propto \pi_\varepsilon(y|\boldsymbol{x})\pi_{pr}(\boldsymbol{x})$$

Assume: Gaussian prior and likelihood:

$$\boldsymbol{x}_{pr} \sim \mathcal{N}(\boldsymbol{x}_o, L_x^T L_x), \quad \varepsilon \sim \mathcal{N}(0, L_\varepsilon^T L_\varepsilon)$$

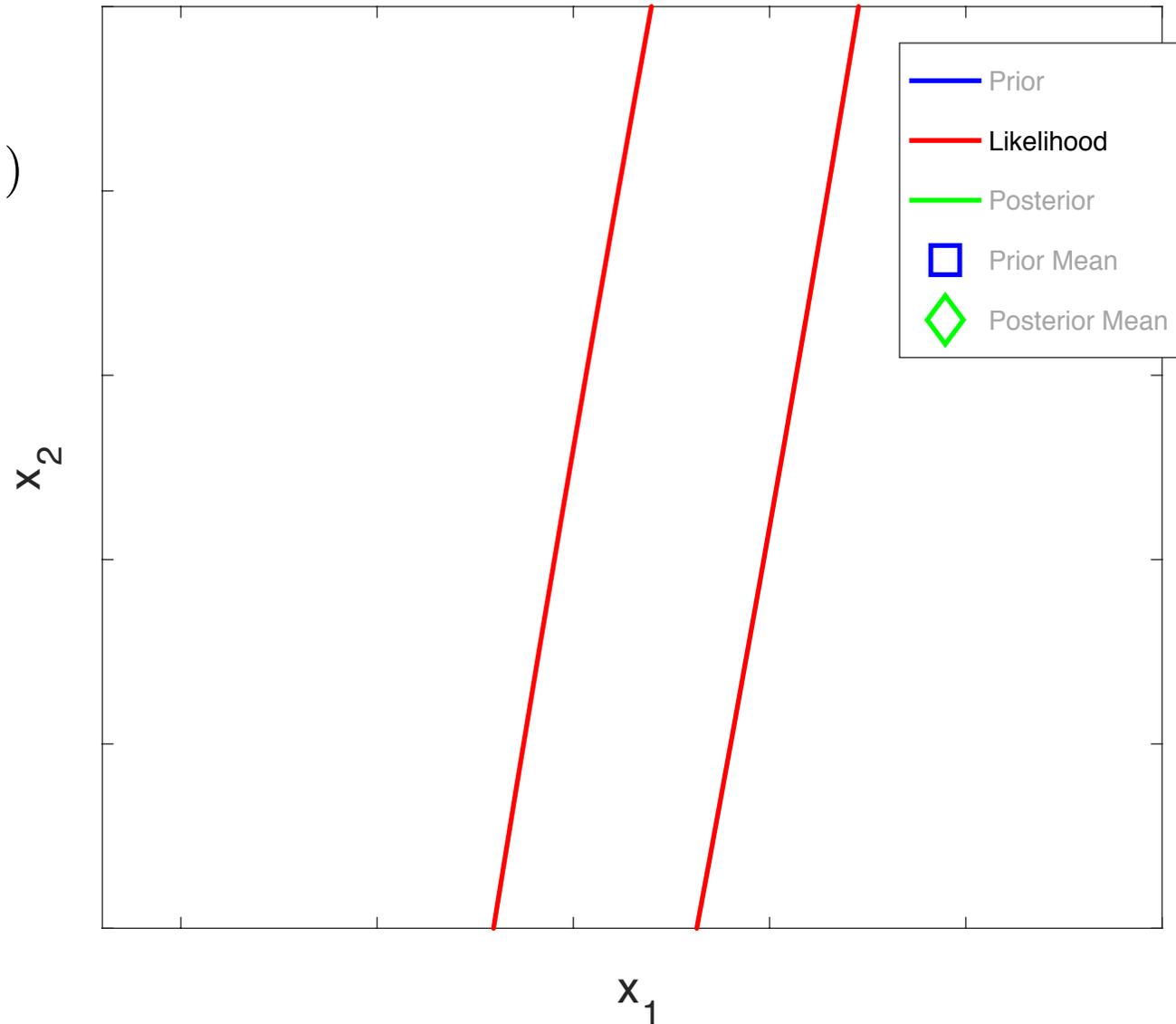




2D Example Posterior

$$\pi_{\varepsilon}(y|x)$$

$$\varepsilon \sim \mathcal{N}(0, L_{\varepsilon}^T L_{\varepsilon})$$





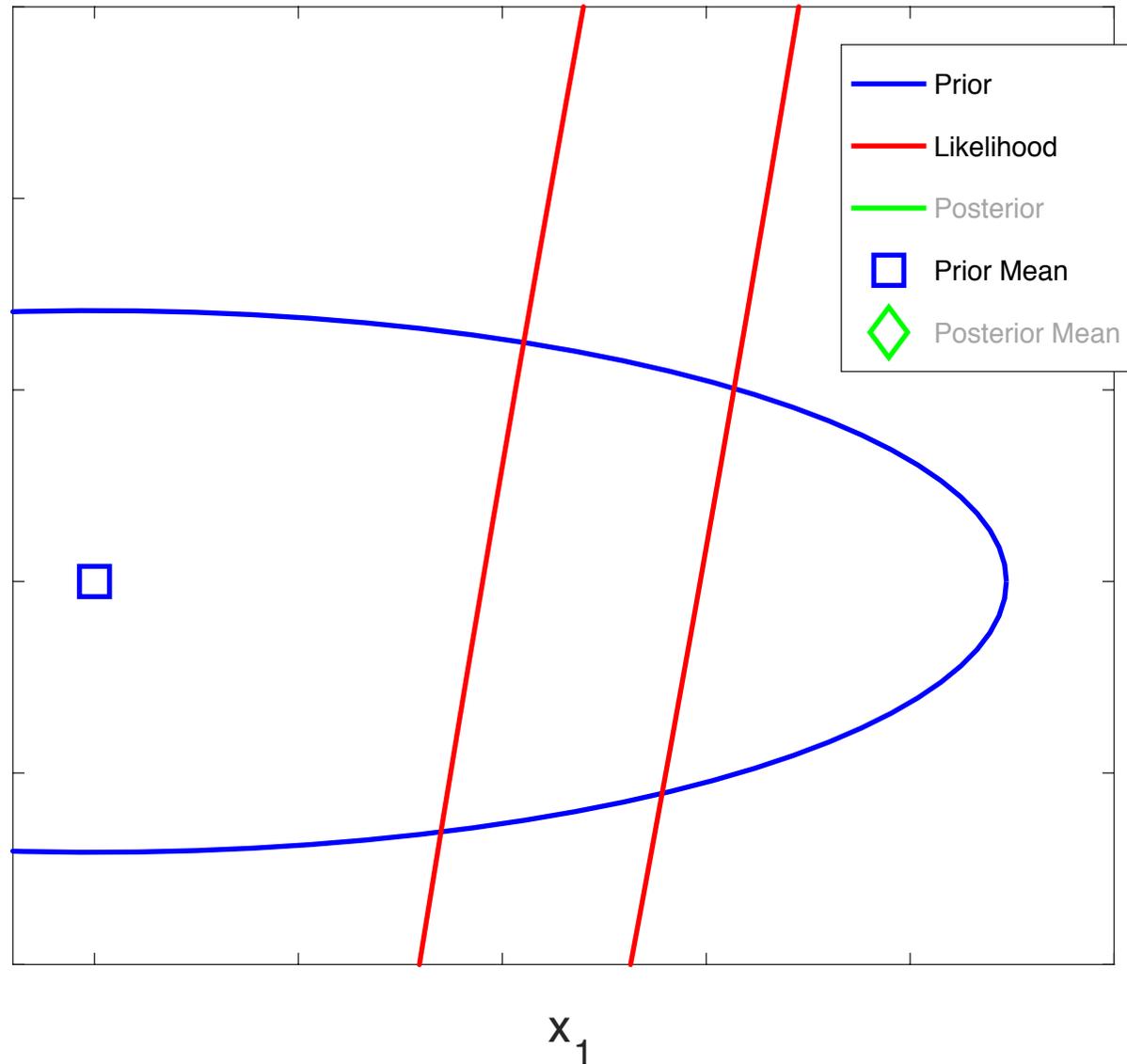
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2D Example Posterior

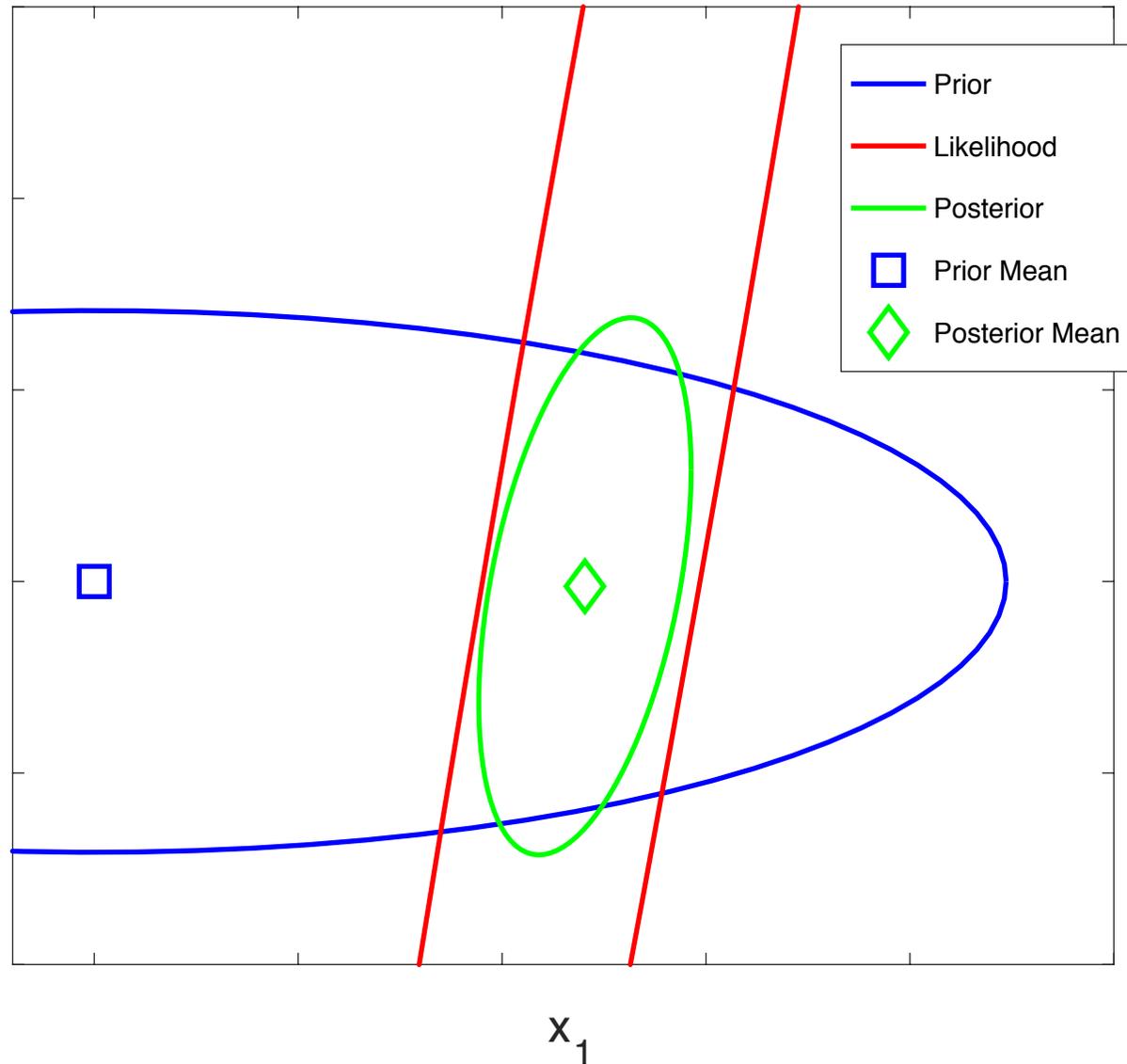
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2D Example Posterior

Informative directions [3]

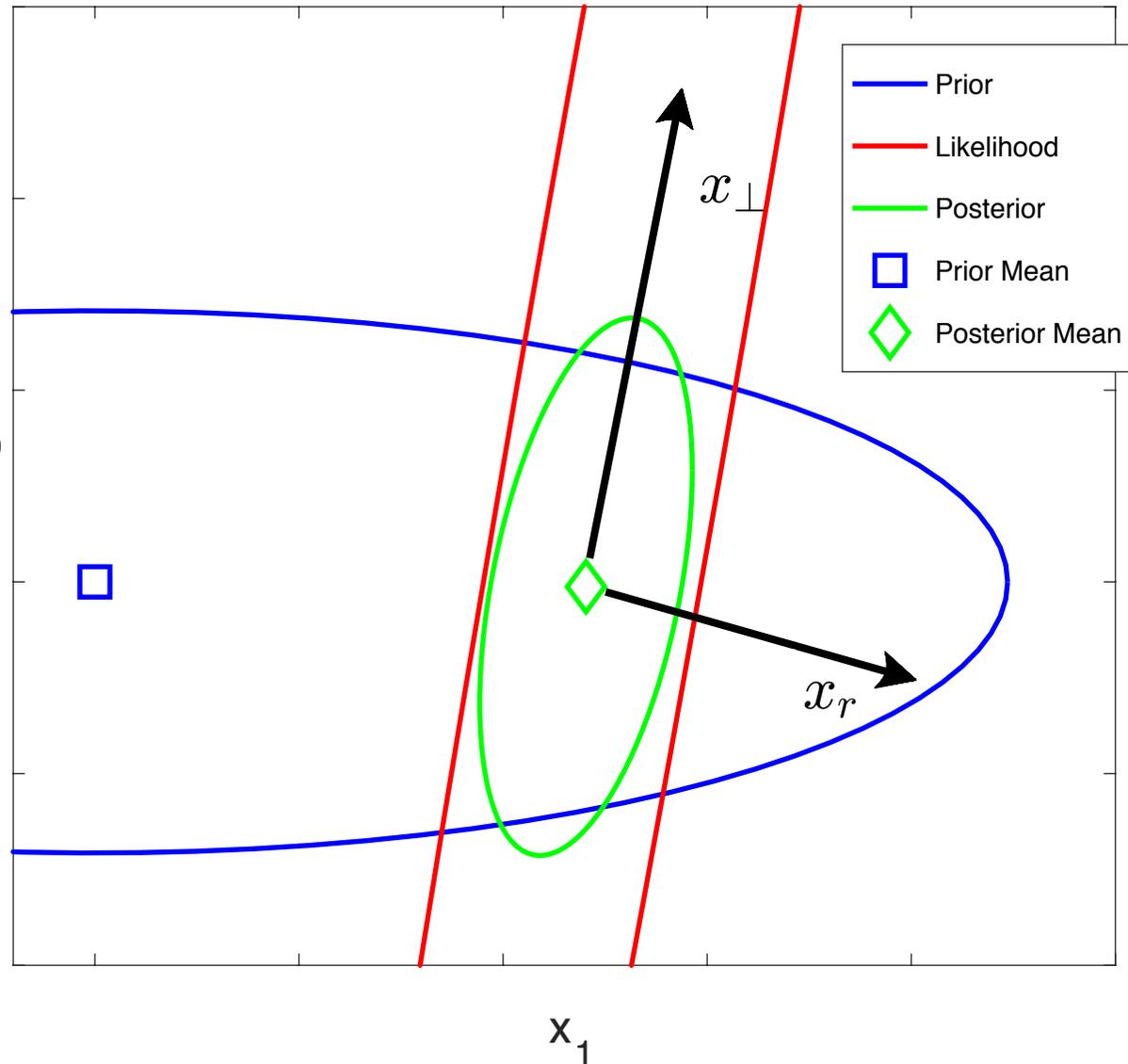
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Likelihood-Informed (LIS) Dimension Reduction

Following T. Cui & al. [4] we use SVD $\tilde{J}\tilde{J}^T = U\Lambda V^T$ and define matrices

$$\Phi_r = L_x V_{1:r}, \quad \Phi_{\perp} = L_x V_{r+1:N},$$



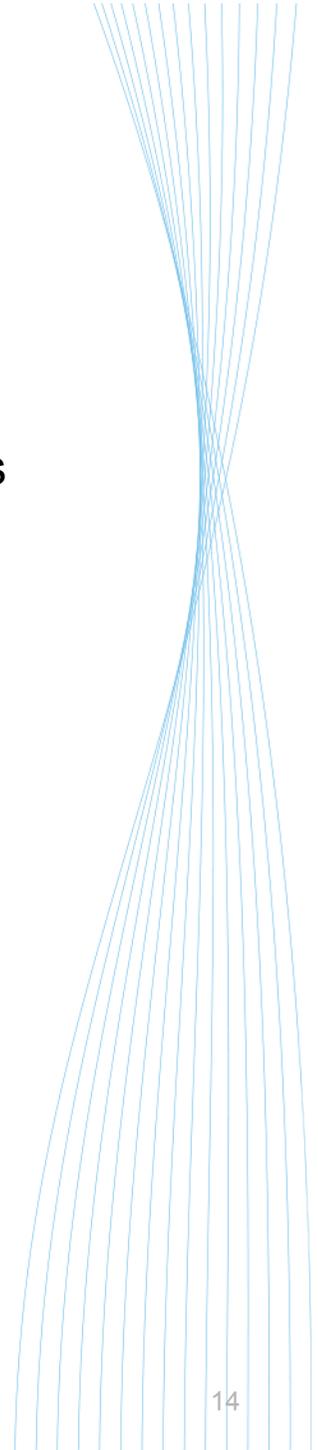
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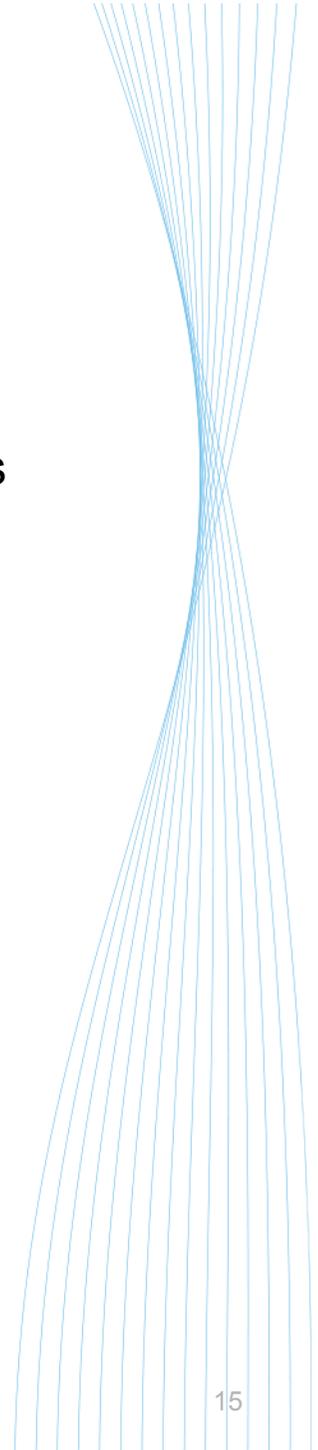
$$\Phi_r = L_x V_{1:r}, \quad \Phi_{\perp} = L_x V_{r+1:N},$$

we can decompose \boldsymbol{x} as

$$\boldsymbol{x} = \Phi_r \boldsymbol{x}_r + \Phi_{\perp} \boldsymbol{x}_{\perp}$$

The approximate posterior can now be written as

$$\tilde{\pi}(\boldsymbol{x}|\boldsymbol{y}) \propto \pi(\boldsymbol{y}|\Phi_r \boldsymbol{x}_r) \pi_r(\boldsymbol{x}_r) \pi_{\perp}(\boldsymbol{x}_{\perp})$$

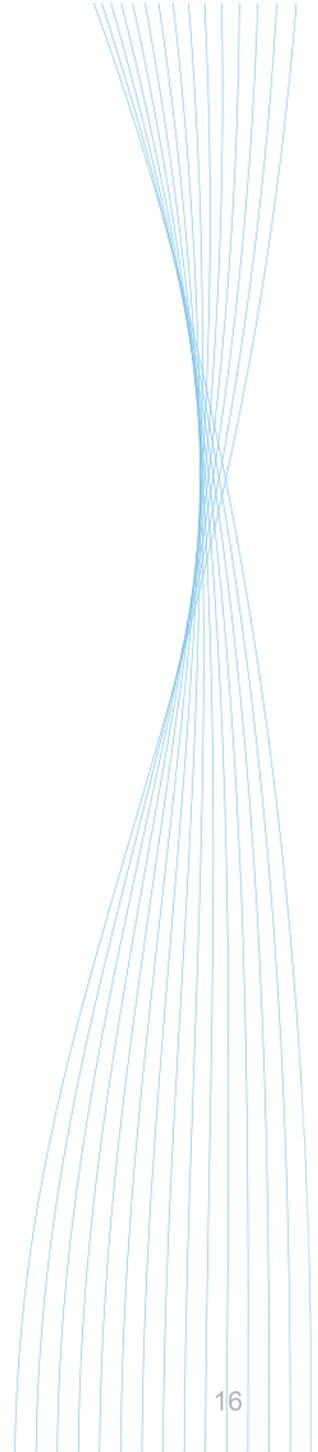




Profile retrieval

- Solution by Optimal Estimation (OE): Maximum A Posteriori estimate by

$$x_{MAP} = \arg \min_{x \in \mathbb{R}^n} \left\{ \|y - F(x)\|_{\varepsilon}^2 + \|x - x_0\|_{pr}^2 \right\}$$





Profile retrieval

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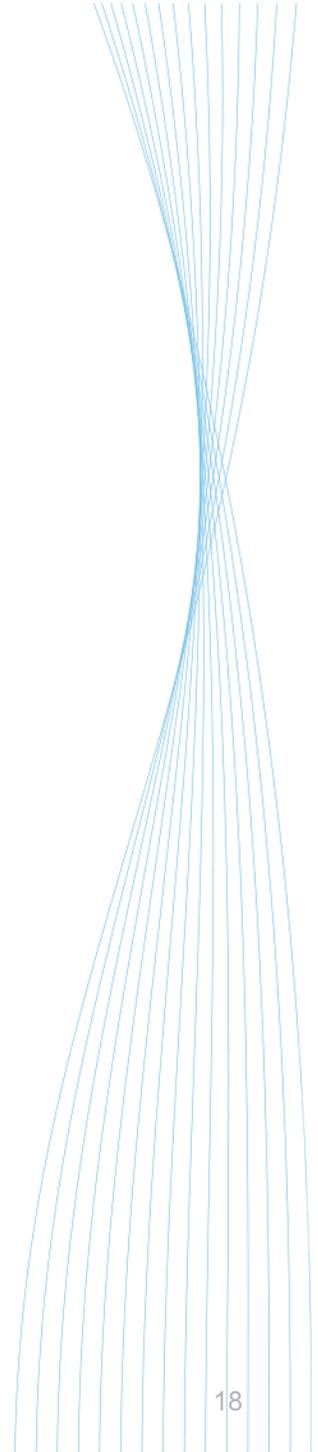
$$x_{MAP} = \arg \min_{x \in \mathbb{R}^n} \left\{ \|y - F(x)\|_{\varepsilon}^2 + \|x - x_0\|_{pr}^2 \right\}$$

- Uncertainty Quantification using Adaptive Markov Chain Monte Carlo (MCMC) **[5]**, significant computational gains with LIS **[6]**



SWIRLAB

Freely available MATLAB toolbox by Simo Tukiainen (FMI) **[7]**

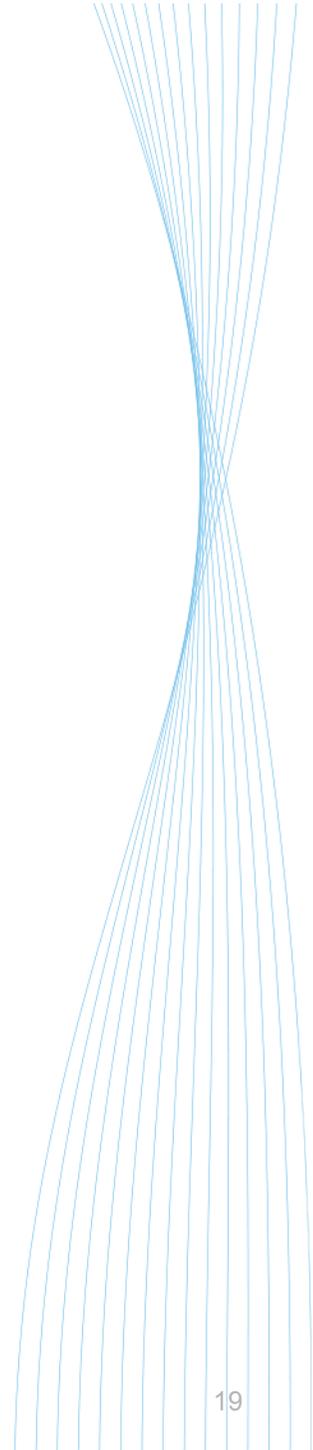




SWIRLAB

Freely available MATLAB toolbox by Simo Tukiainen (FMI) [7]

- Radiative transfer forward model for FTS retrieval

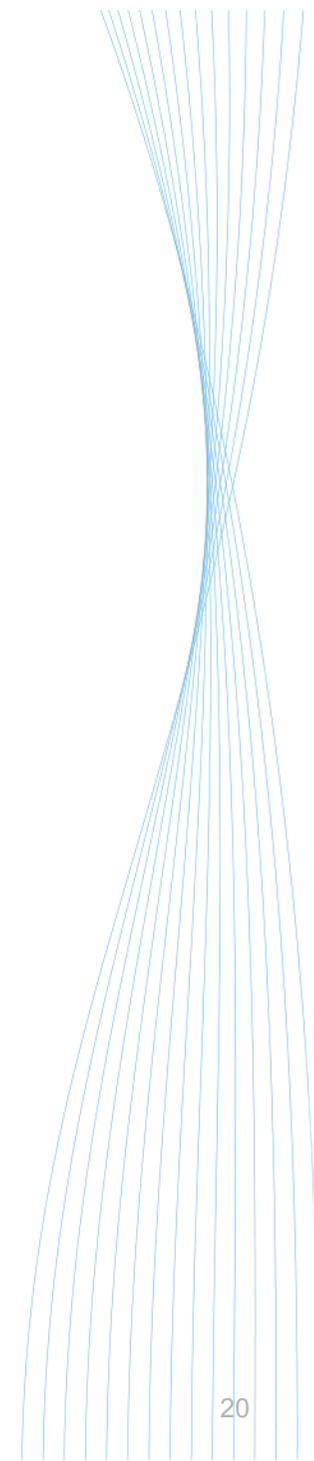




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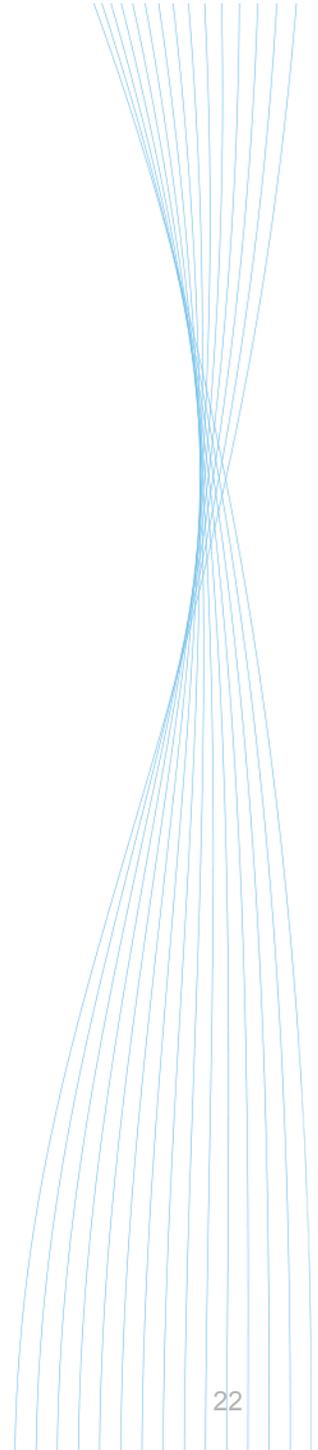
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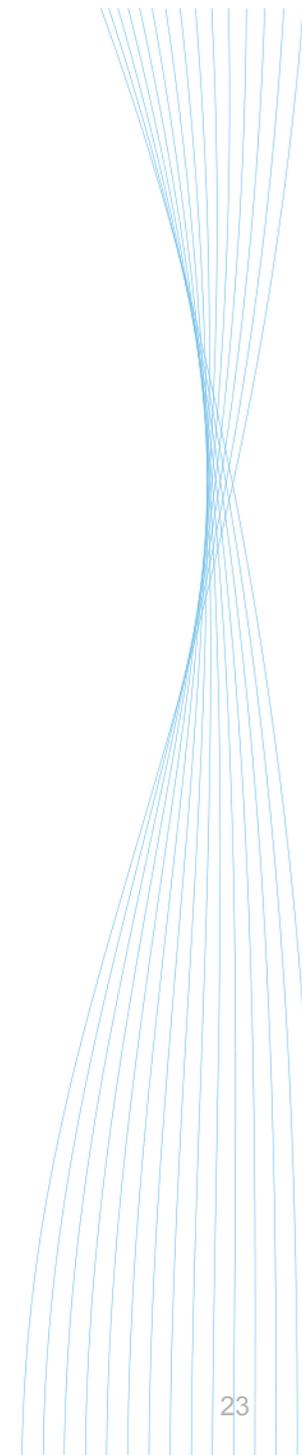




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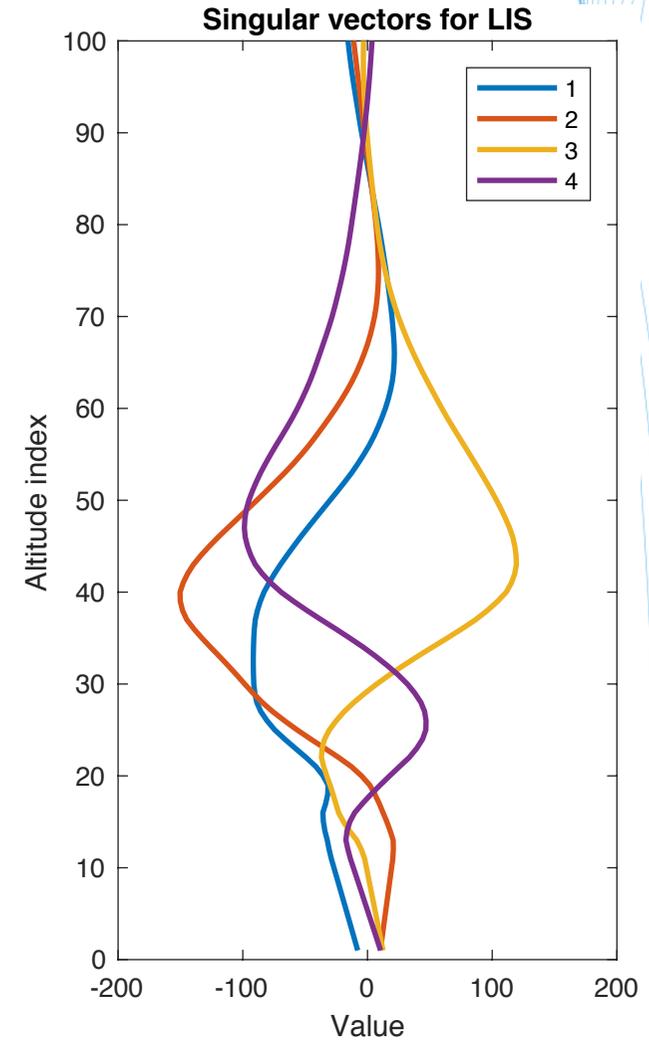
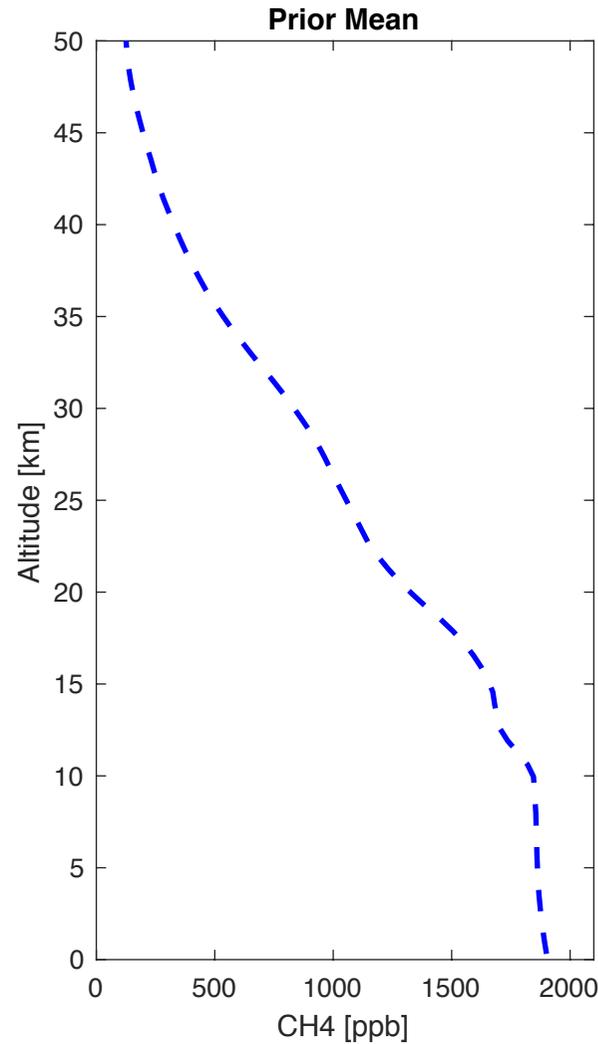
- Radiative transfer forward model for FTS retrieval
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- Temperature, pressure and solar spectrum from GGG2014
- Retrieval using LIS dimension reduction: OE & MCMC





SWIRLAB profile retrieval

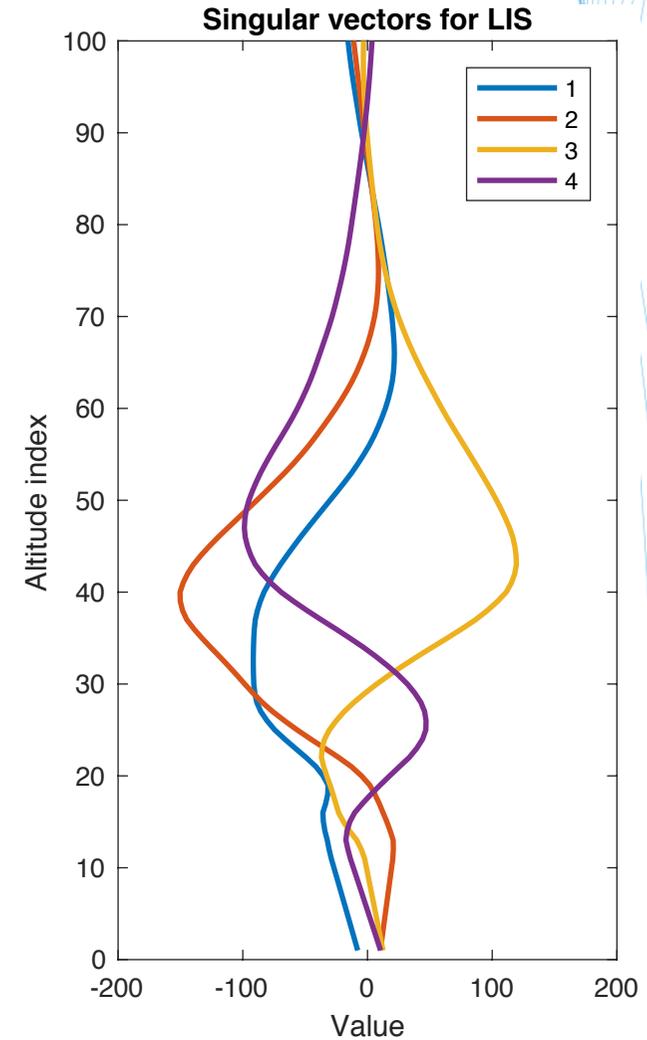
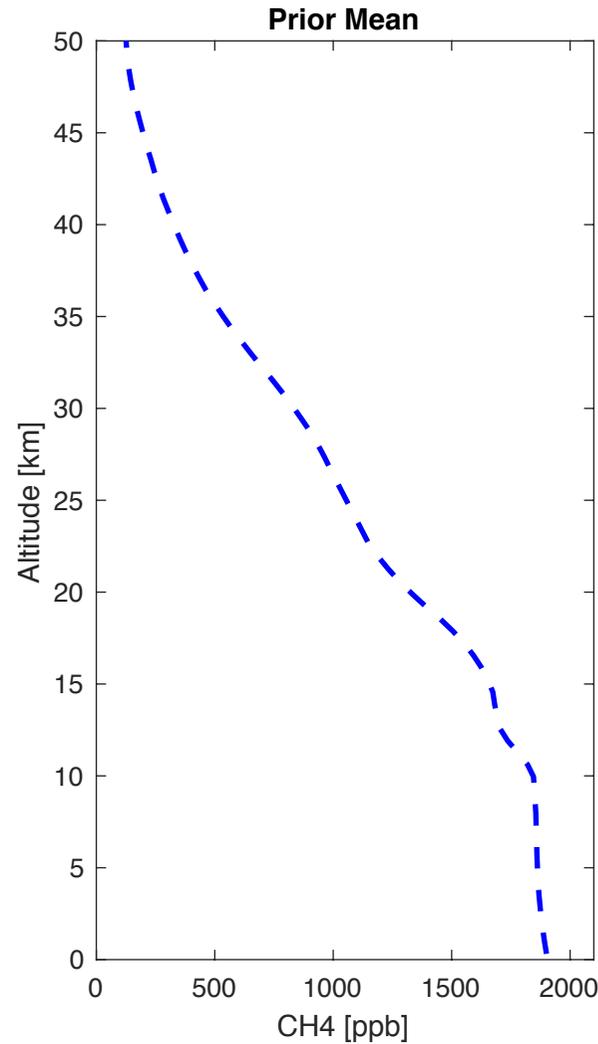
- Optimal Estimation based fast retrieval algorithm for the FTS problem





SWIRLAB profile retrieval

- Optimal Estimation based fast retrieval algorithm for the FTS problem
- Motivation: currently, operational retrieval only has 1 degree of freedom: scaling the prior mean

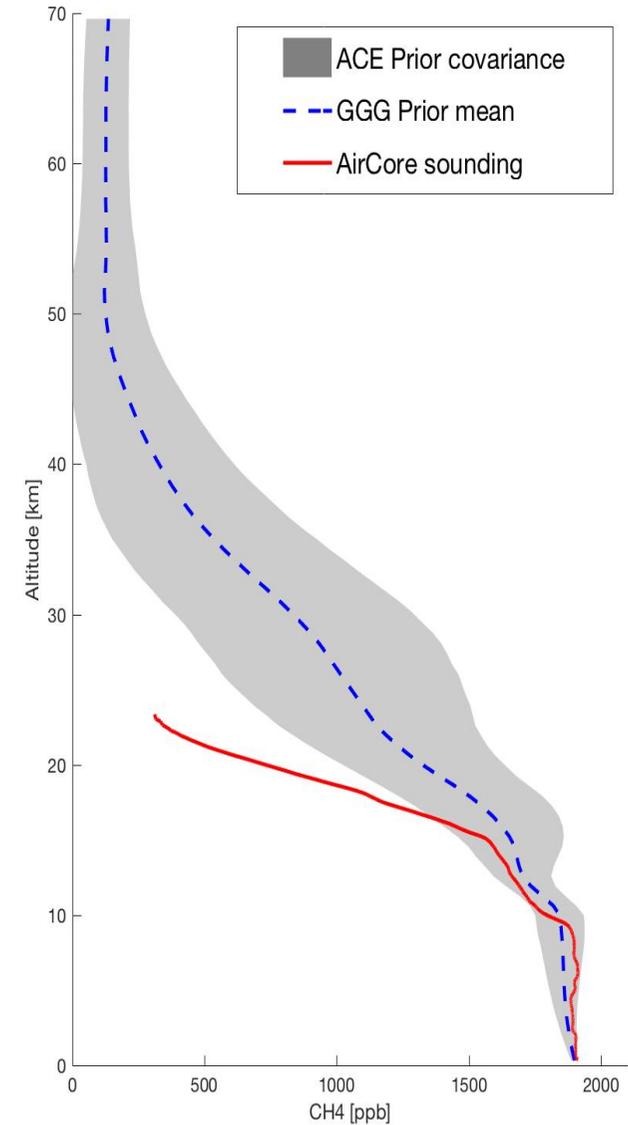
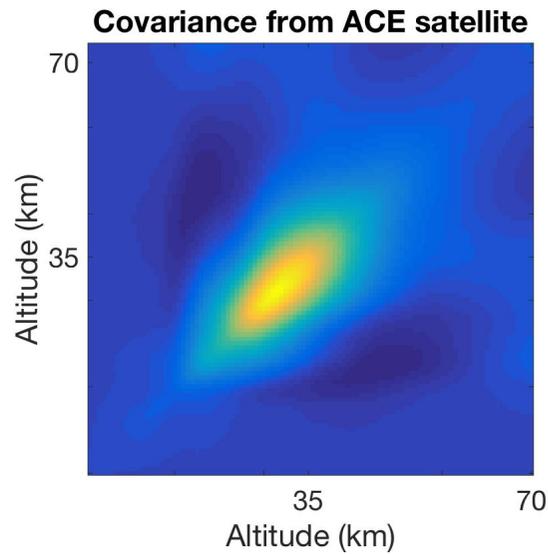




SWIRLAB Prior

Multivariate Gaussian:

Covariance: derived from an ensemble of ACE-FTS satellite measurements.





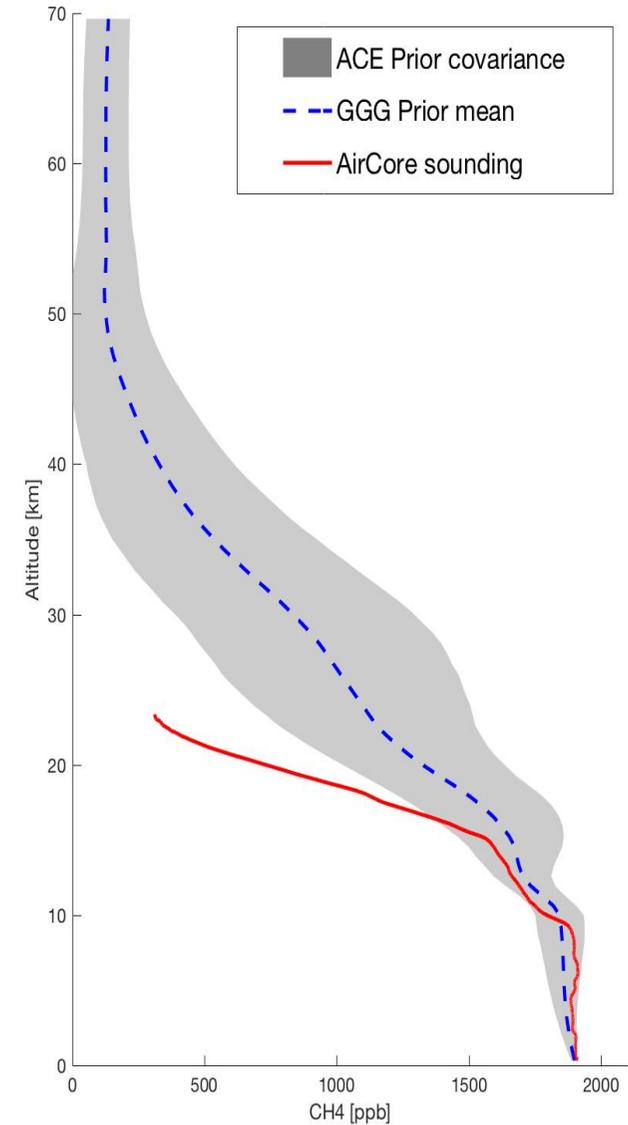
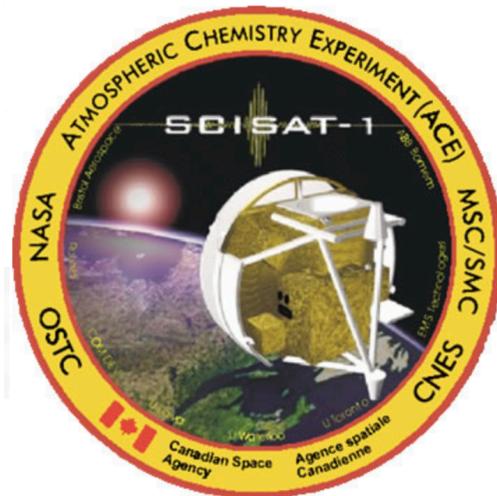
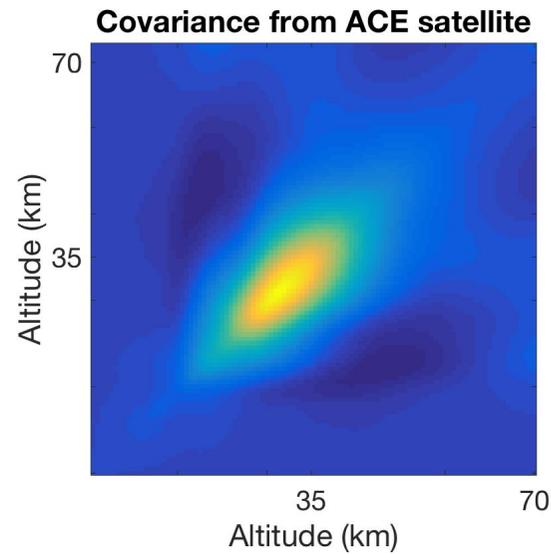
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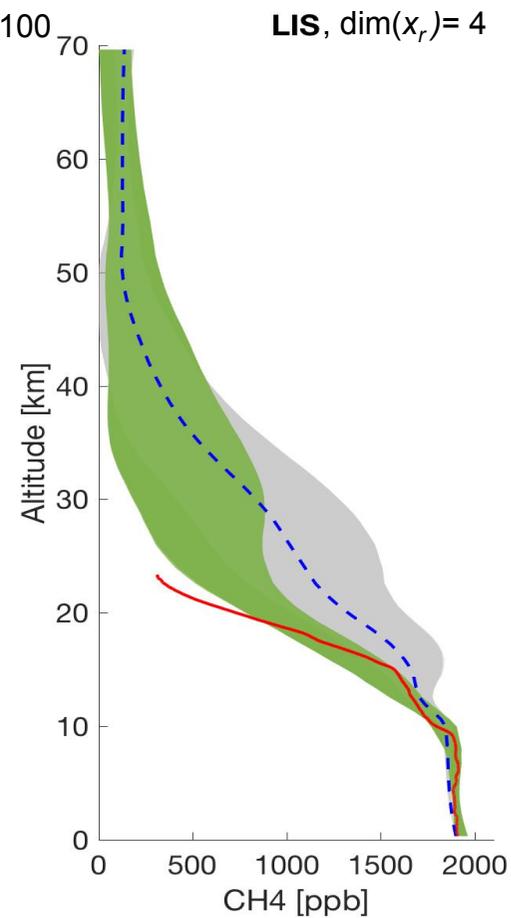
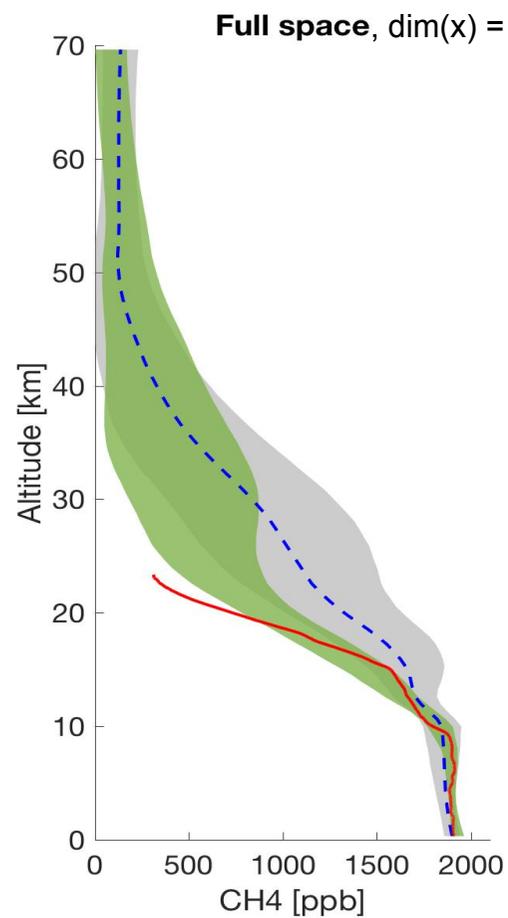
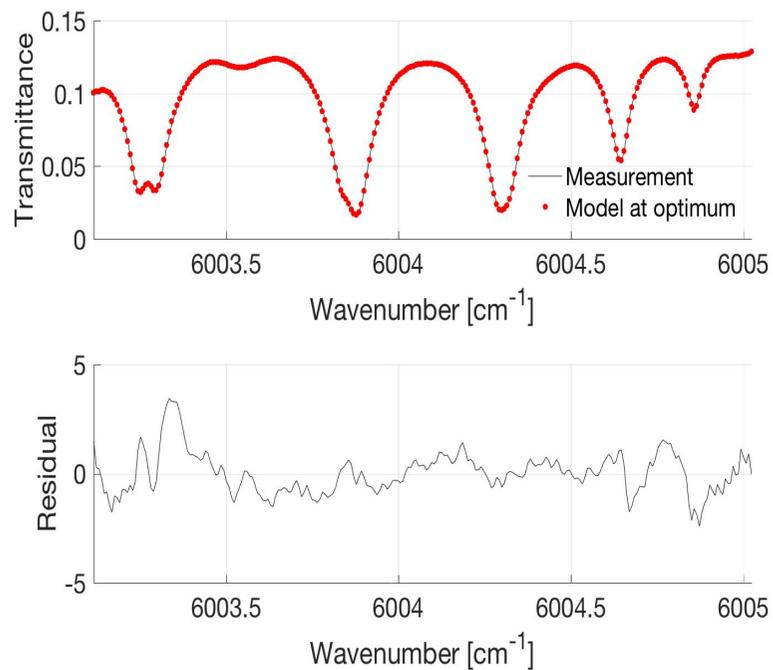
Mean:

- Lower part from GGG2014
- Upper part from ACE-FTS



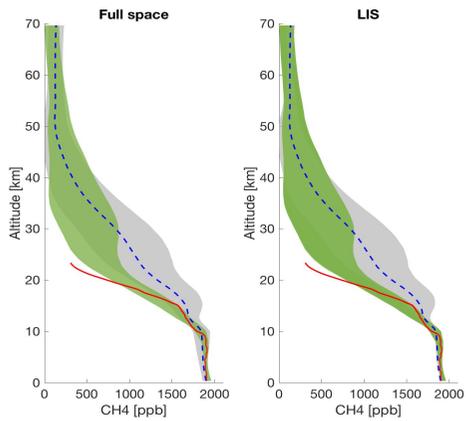


MCMC results



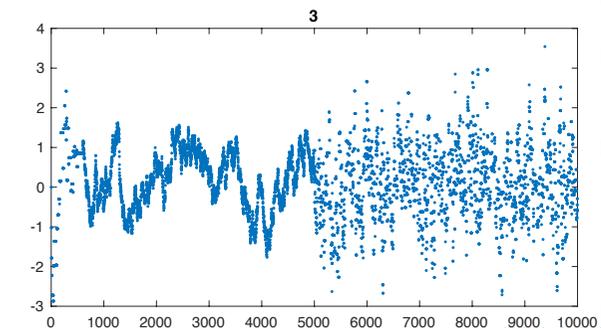
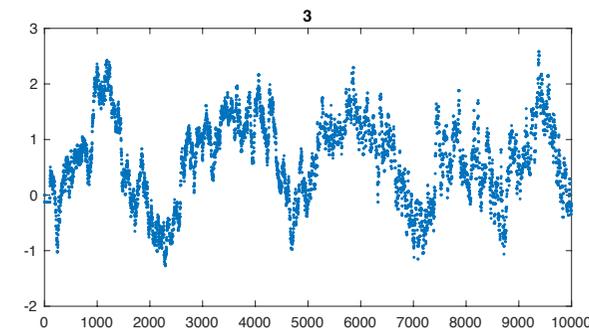
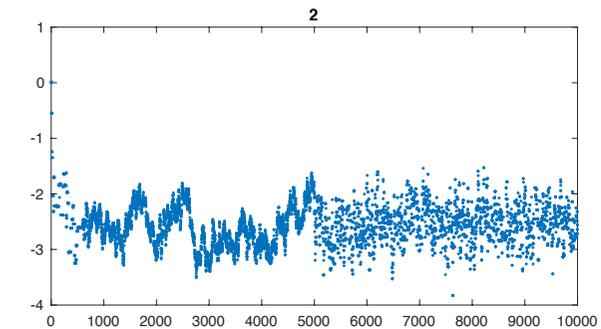
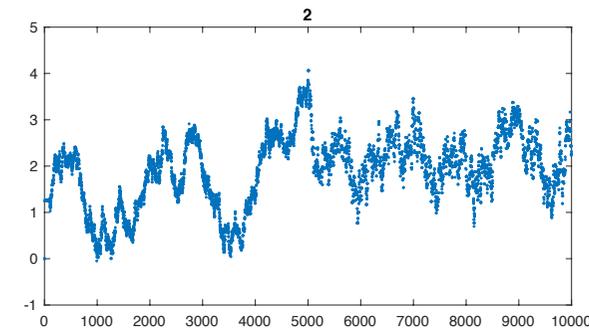
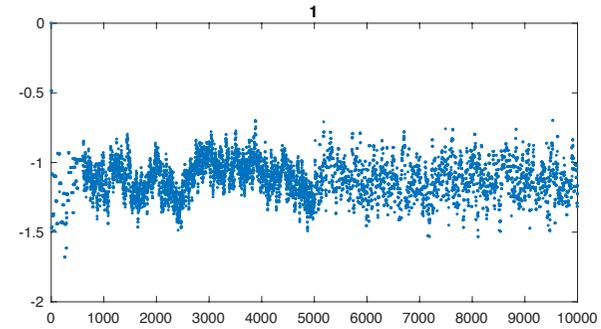
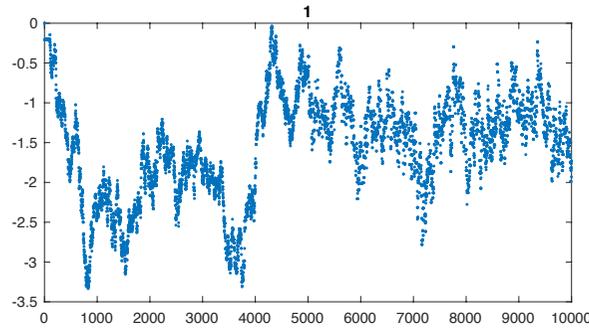


MCMC results



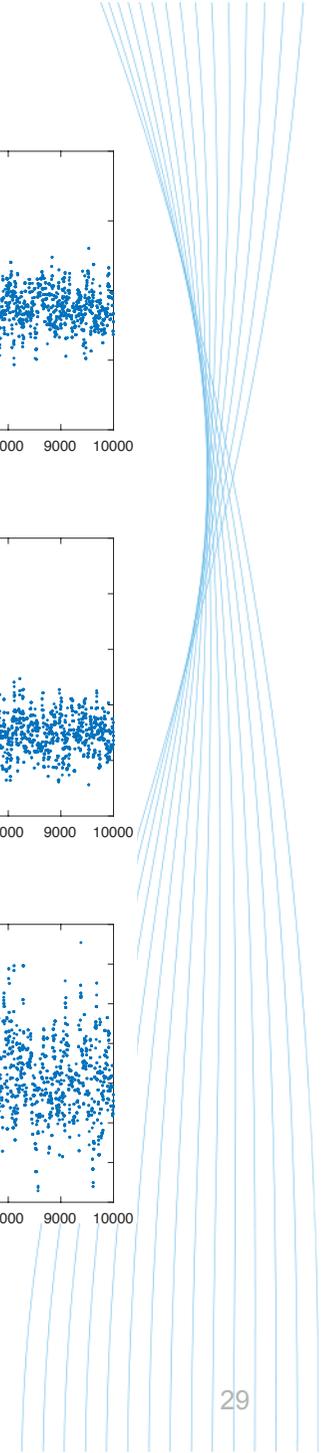
$$\mathbb{V}(\text{full}) = 1,56$$

$$\mathbb{V}(\text{LIS}) = 19,01$$



Full space, dim = 100

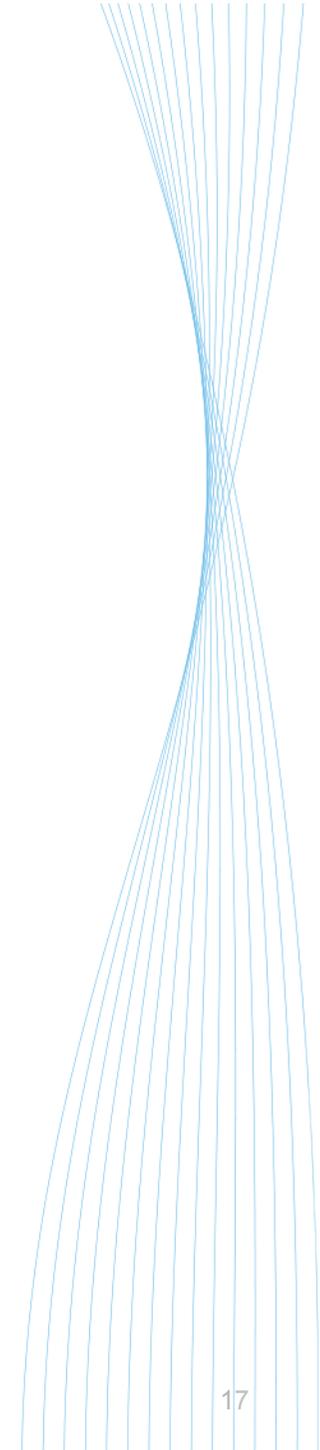
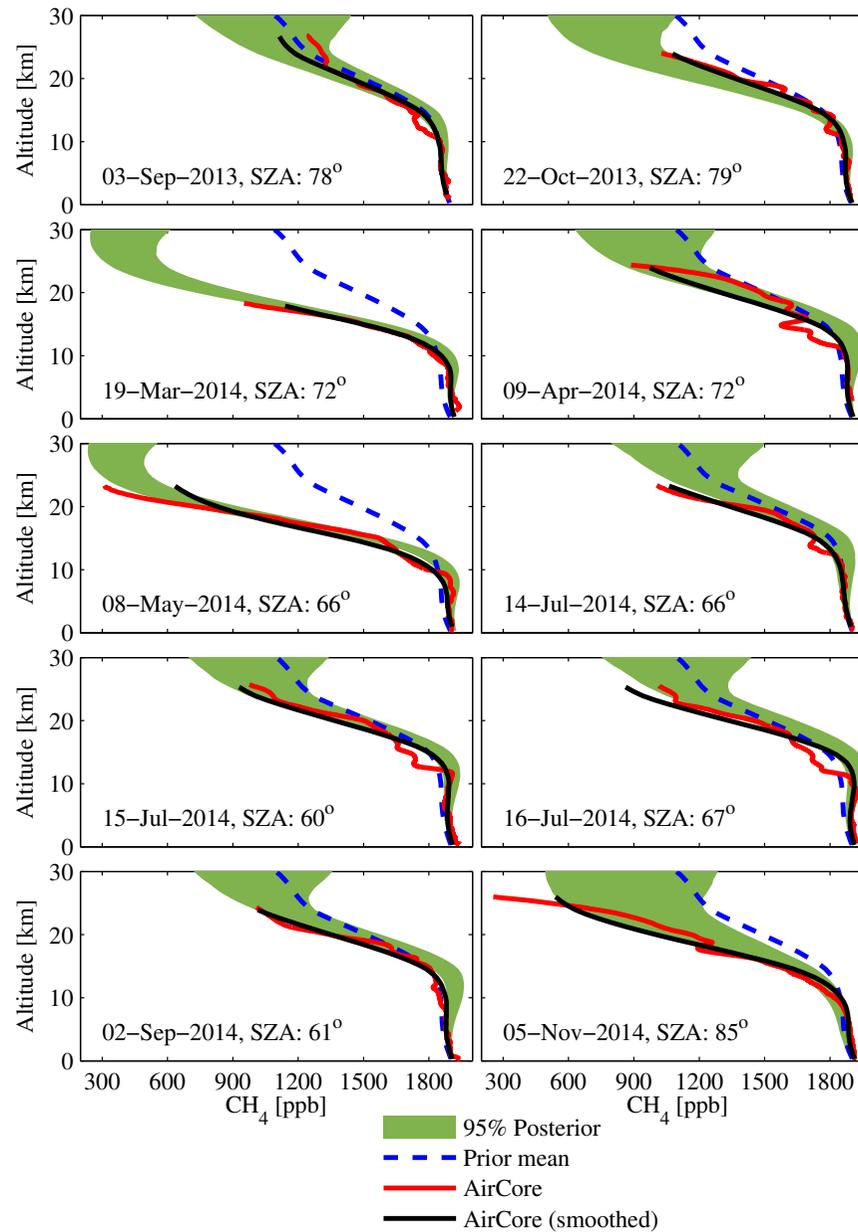
LIS, dim = 4





MCMC results

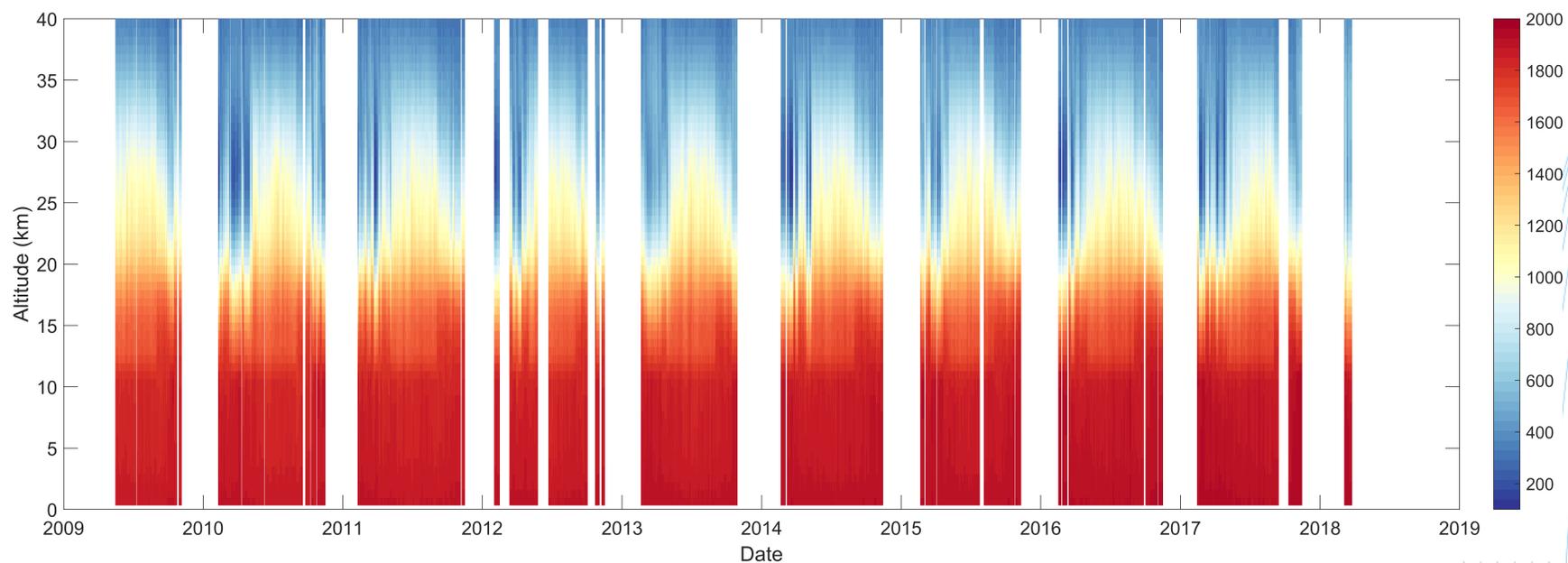
Retrievals for several measurements:



Retrieval of time series: 2009-2017

Preliminary results: vertical information on CH₄

- allows time series analysis on different altitudes

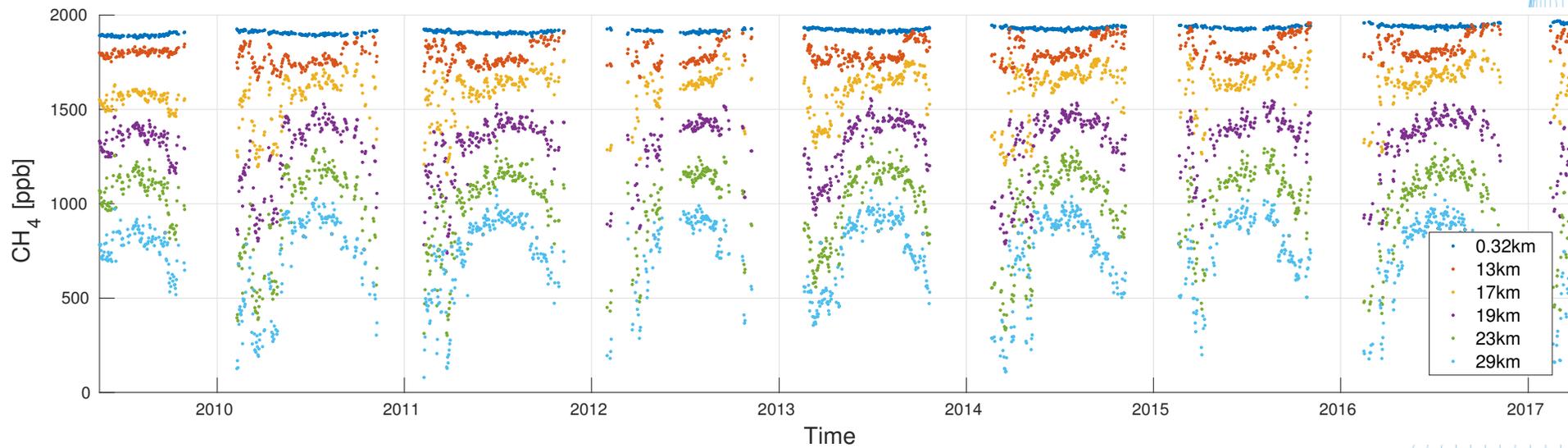




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Dynamic Linear Model (DLM)

Smooth regression tool for time series analysis, MATLAB toolbox by Marko Laine [8]



Dynamic Linear Model (DLM)

Smooth regression tool for time series analysis, MATLAB toolbox by Marko Laine [8]

- Hierarchical statistical model for uncertainties in data, process and parameters

$$y_t = F_t x_t + v_t \quad v_t \sim N(0, V_t)$$

$$x_t = G_t x_{t-1} + w_t \quad w_t \sim N(0, W_t)$$

- Can be used to extract trend, seasonal component etc.

y_t : observations

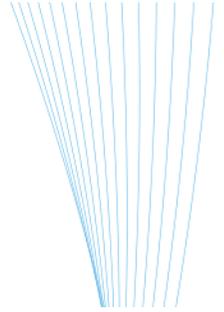
x_t : hidden model states

F_t : observation operator

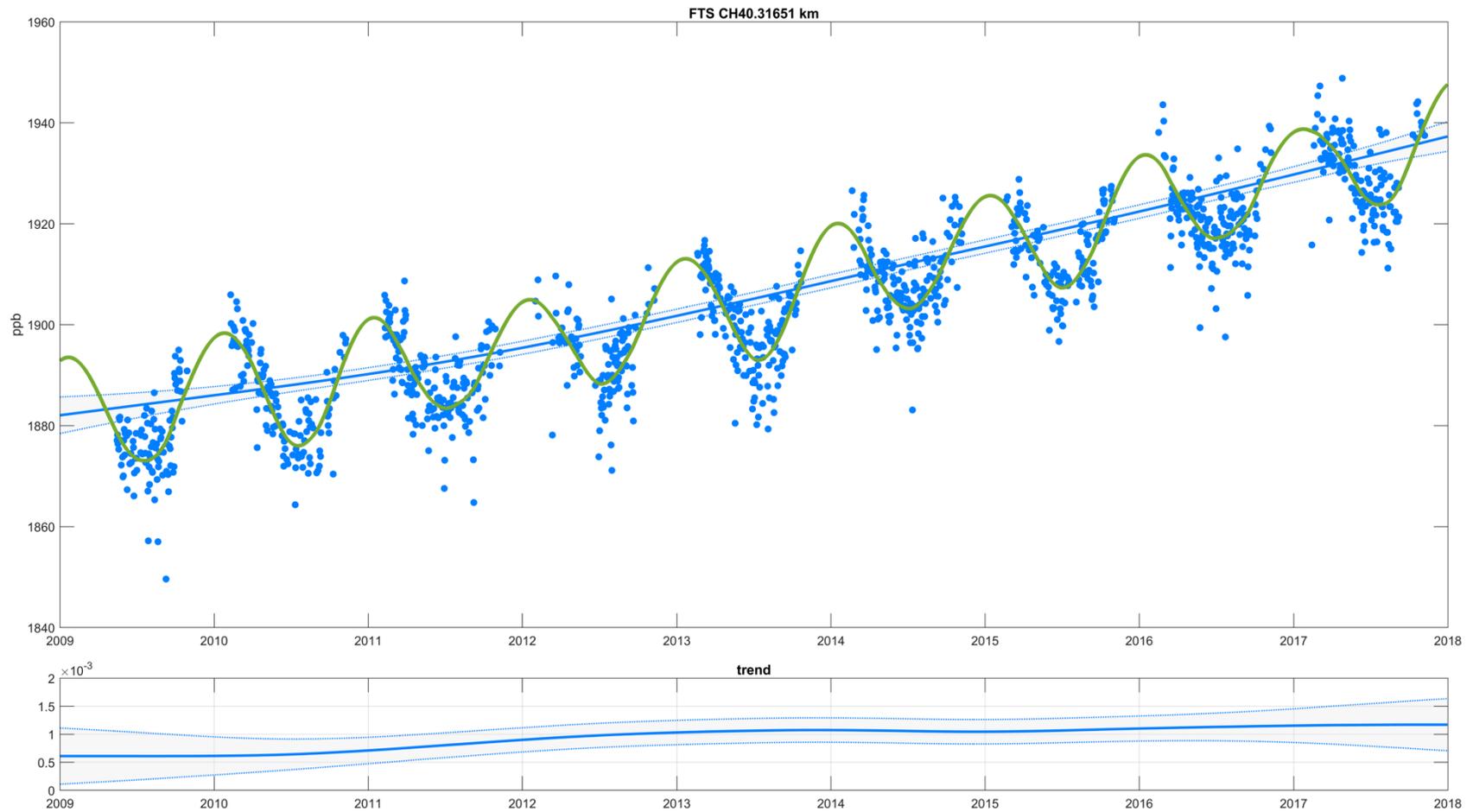
G_t : model operator

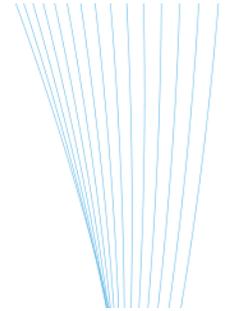
v_t : observation uncertainty

w_t : model uncertainty

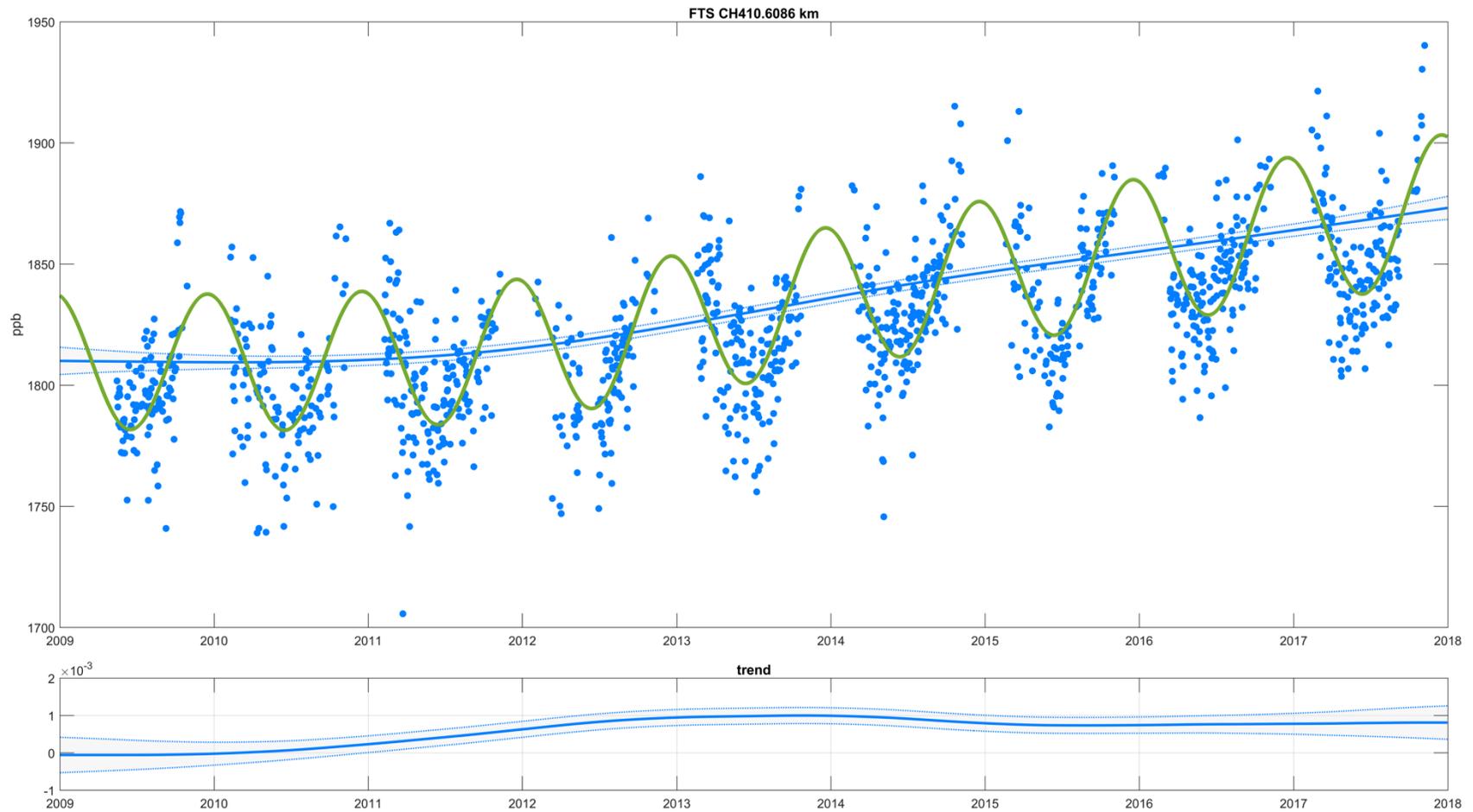


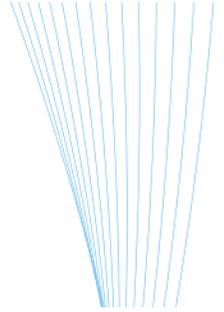
DLM fit: FTS (0.3km)



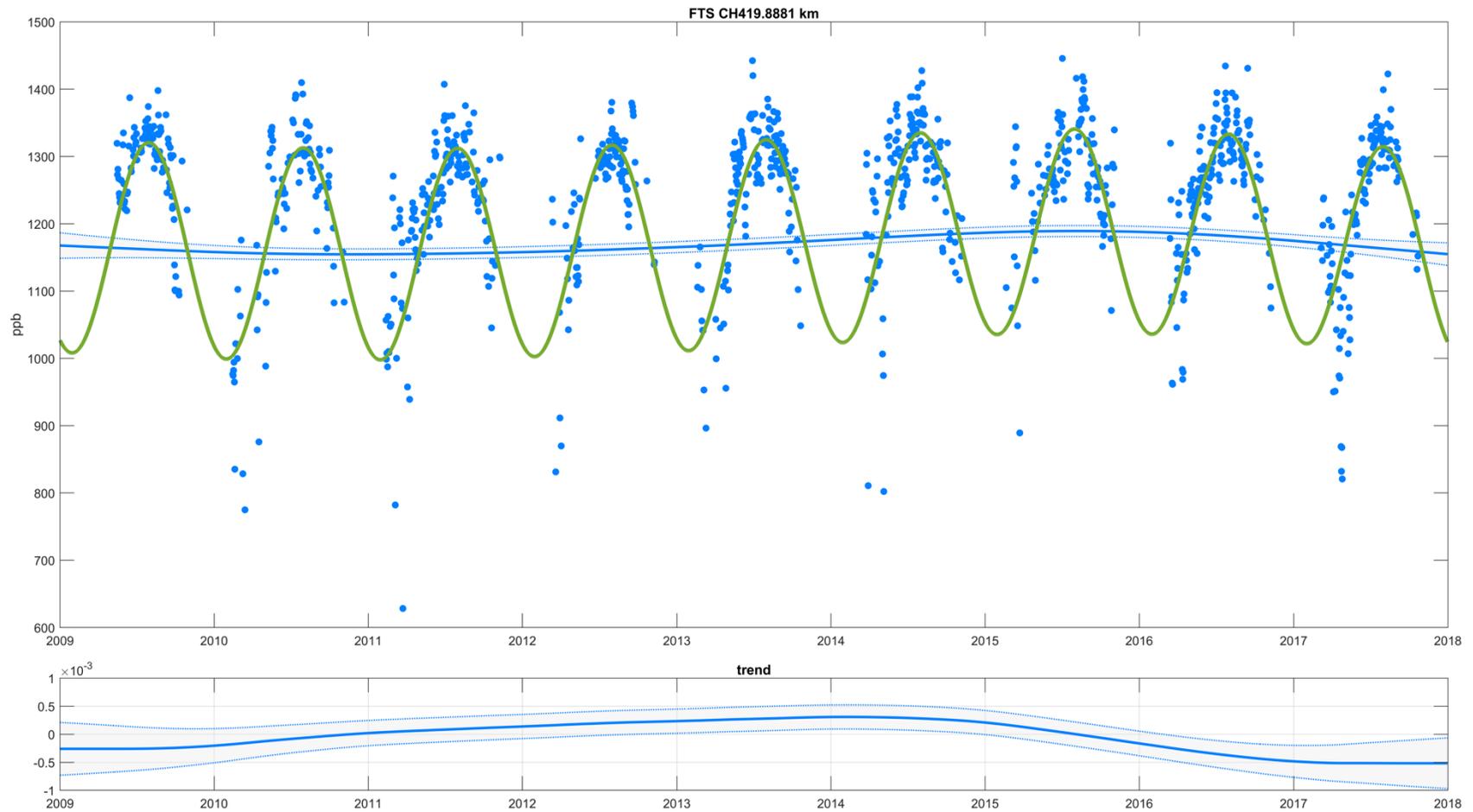


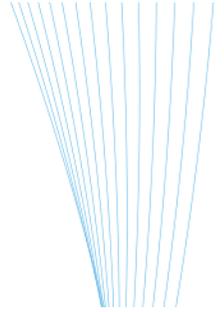
DLM fit: FTS (10km)



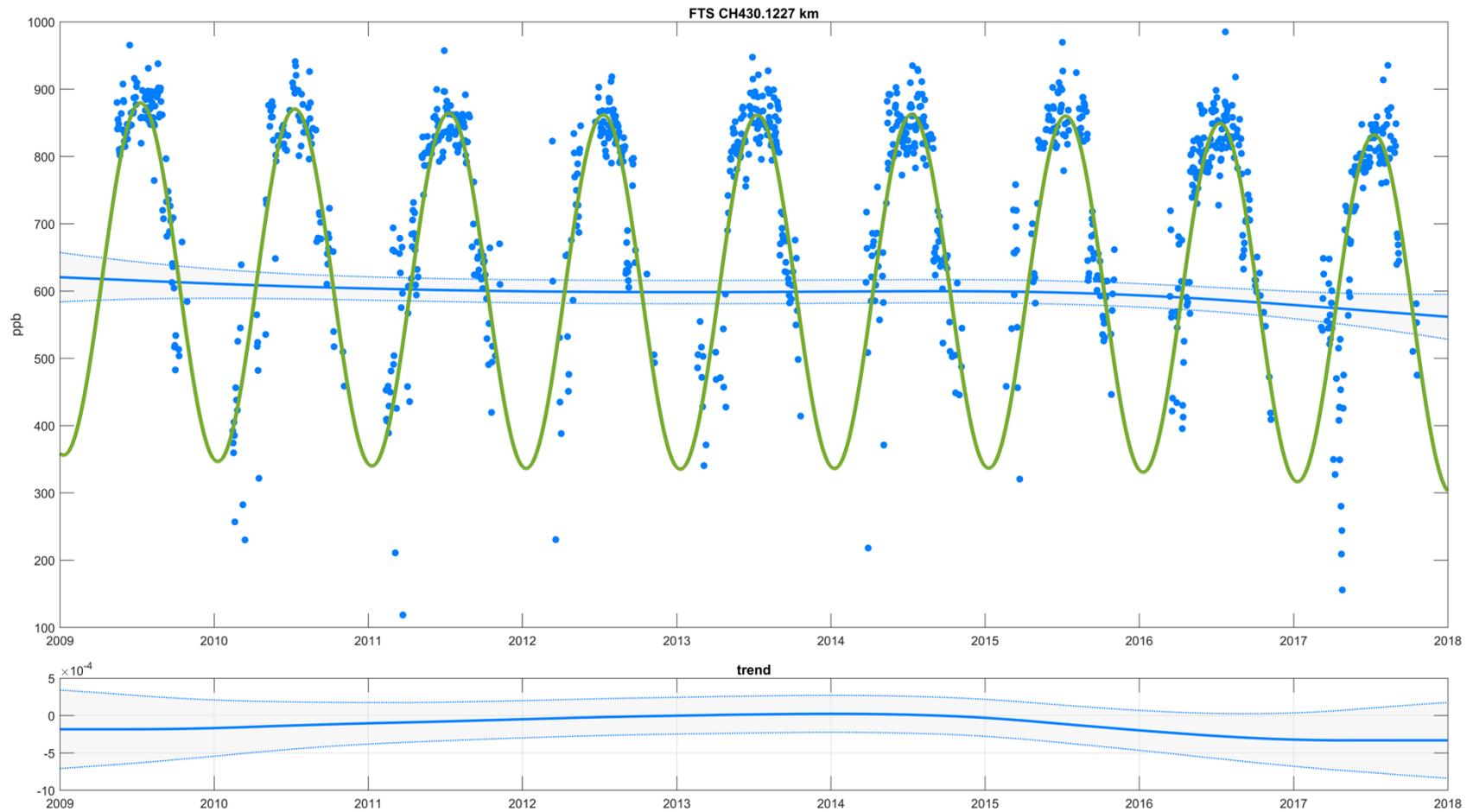


DLM fit: FTS (20km)



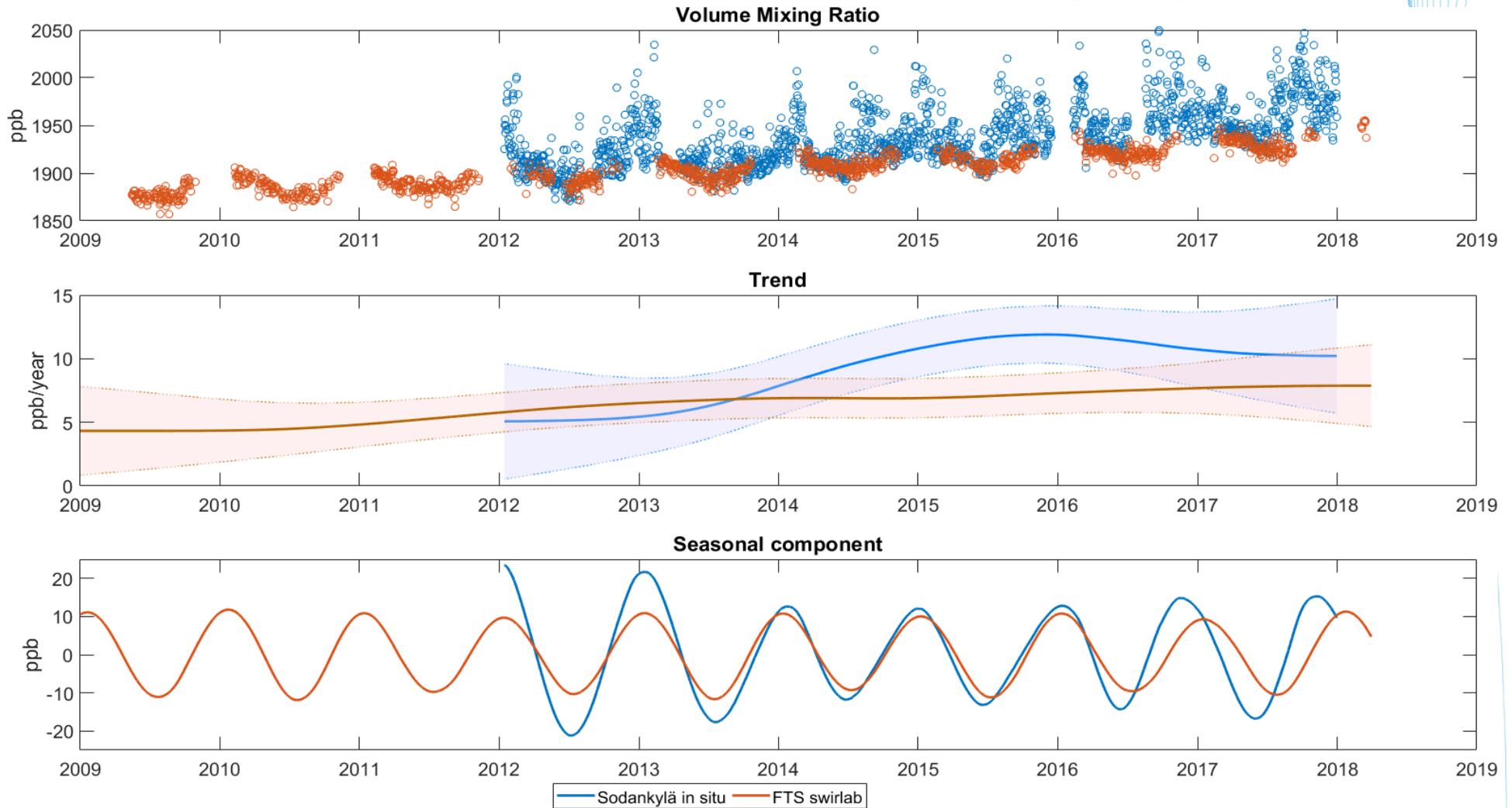


DLM fit: FTS (30km)



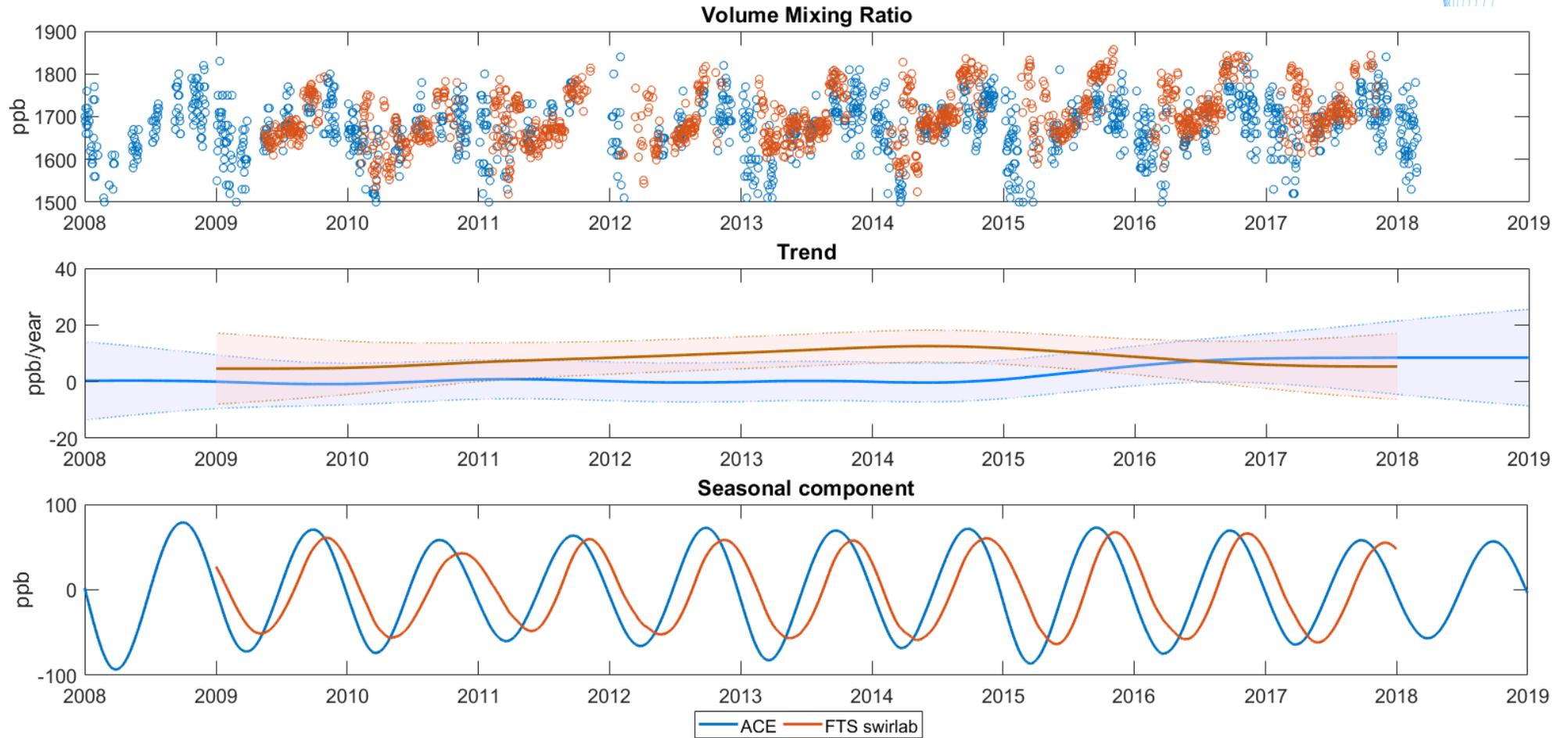


Comparison: **FTS** (300m) vs. Sodankylä in situ (50m)





Comparison: **FTS** vs ACE (13.5km)





References

- [1] R. Kivi, P. Heikkinen: Fourier Transform Spectrometer measurements of column CO₂ at Sodankylä, Finland. *Geoscientific Instrumentation, Methods and Data Systems* 5(2):271-279, 2016
- [2] Karion, A., Sweeney, C., Tans, P., Newberger, T.: AirCore: An Innovative Atmospheric Sampling System. *Journal of Atmospheric and Oceanic Technology* 27(11), 1839–1853 (2010). DOI 10.1175/2010JTECHA1448.1
- [3] Rodgers C.D., *Inverse Methods for Atmospheric Sounding: Theory and Practice*, World Scientific Publishing Co. Pte. Ltd., 2000.
- [4] Cui T., Martin J., Marzouk Y., Solonen A., Spantini A., *Likelihood-informed dimension reduction for nonlinear inverse problems*, *Inverse Problems*, 30 (2014), p 114015.
- [5] Haario, H., Saksman, E., Tamminen, J.: An adaptive Metropolis algorithm. *Bernoulli* 7(2), 223–242 (2001). DOI 10.2307/3318737
- [6] O. Lamminpää, M. Laine, S. Tukiainen, J. Tamminen: Likelihood informed dimension reduction for inverse problems in remote sensing of atmospheric constituent profiles <https://arxiv.org/abs/1709.02611v1>
- [7] SWIRLAB: <https://github.com/tukiains/swirlab>
- [8] DLM: <http://helios.fmi.fi/~lainema/dlm/>



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Thank you!

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08/05/18

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